



UiT The Arctic University of Norway

UiT School of Business and Economics  
Working Papers in Economics No. 2/21

Partial Information Disclosure in a Contest  
Derek J. Clark and Tapas Kundu



Centre for Economic Research



UiT School of Business and Economics

Working Paper in Economics

Number 02/21

March 2021

Partial Information Disclosure in a Contest

Derek J. Clark

School of Business and Economics

UiT - The Arctic University of Norway

NO-9037 Tromsø, Norway

[derek.clark@uit.no](mailto:derek.clark@uit.no)

and

Tapas Kundu

Oslo Business School

Oslo Metropolitan University

Pilestredet 35

NO-0166 Oslo, Norway

[tapas.kundu@oslomet.no](mailto:tapas.kundu@oslomet.no)

© Derek J. Clark & Tapas Kundu.

This is an Open Access Article distributed under the terms of the Creative Commons Attribution License (<https://creativecommons.org/licenses/by/4.0/legalcode>).

<https://doi.org/10.7557/22.5775>

ISSN: 2535-6534

## Abstract

Zhang and Zhou [2016] use the concept of Bayesian persuasion due to Kamenica and Gentzkow [2011] to analyze information disclosure in a contest with one-sided asymmetric information. They show that an effort-maximizing designer can manipulate information disclosure to increase expected efforts in the contest, but base their analysis upon active participation in the contest by all types of the informed player. We extend their analysis to equilibria in which some informed types exert no effort in the contest, showing how this changes the type of information disclosure that arises.

JEL Codes: D02, D72, D82

Keywords: Contest; Information design; Bayesian persuasion

# 1 Introduction

Contests in which resources are sunk to win a prize capture competition in social, political and economic spheres. A common theme is how a designer (principal) can maximize the resources expended in the contest. Recently Zhang and Zhou [2016] introduced information disclosure as an instrument at the disposal of the principal, using the Bayesian Persuasion framework of Kamenica and Gentzkow [2011]. In a two-player contest, Zhang and Zhou [2016] focus on one-sided informational asymmetry, where one player has better information than the competitor and the principal. The effort-maximizing, but uninformed, principal initially commits to a set of state-conditional distribution of signals before realization of the state, which is the value of the prize to the player with private information; the signals disclose all or no information at two extremes, but can also impart a particular posterior belief to the uninformed. The optimal distribution of signals raises the principal's payoff to the concavification of the total expected effort function.

Zhang and Zhou [2016] show first that binary values for the state yields an expected effort function that is either globally convex or concave; in the former case, full information disclosure is optimal, and in the latter there is no disclosure.<sup>1</sup> Only when there are more than two possible valuations can partial disclosure appear, in which the signal reveals the true value of the prize imperfectly to the uninformed player. Zhang and Zhou [2016] consider only fully internal solutions in which all types of the informed player have an effort level above zero. Epstein and Mealem [2013] show with two types for the informed player that an equilibrium exists in which the lower value type will not exert effort in the contest. We extend the results of Zhang and Zhou [2016] by considering equilibria in which some types exert no effort, showing that (i) partial disclosure can be optimal even in the two-type case and (ii) that the results on information disclosure are not robust to our extension when there are more than two types.

## 2 Analysis

In Zhang and Zhou [2016], there are two risk-neutral players,  $A$  and  $B$ . Player  $A$ 's value of winning the contest is  $v_A$  and this is common knowledge. Player  $B$ 's value  $v_B$  (the state) is private information, but it is commonly known that it takes  $N \geq 2$  values,  $v_1 < v_2 < \dots < v_N$ , with prior  $\mu^0 = (\mu_1^0, \dots, \mu_N^0) \in P^N = \{(p_1, \dots, p_N) : p_j \geq 0, \sum_{j=1}^N p_j = 1\}$ . Before the state is realized, the contest designer commits to a signaling mechanism, which consists of a family of state-conditional distributions  $\{\Pr[m_s | v_j] \geq 0 : m_s \in S, \sum_{m_s \in S} \Pr[m_s | v_j] = 1\}, j \in \{1, \dots, N\}$  over a finite set of messages  $S$ . We denote the Bayesian posterior after observing message  $m_s \in S$  by  $\mu^s = (\mu_1^s, \dots, \mu_N^s) \in P^N$ . We use the notation  $\mu \in P^N$  to represent any generic distribution over the state space.

In the posterior contest, players exert non-recoverable effort  $(x_A, x_B)$ , which gives player  $i \in$

---

<sup>1</sup>This follows Kamenica and Gentzkow [2011], and is explained later.

$\{A, B\}$  a success probability of

$$p_i(x_A, x_B) = \frac{x_i}{x_A + x_B}.$$

Denote the pure strategy Bayes-Nash equilibrium by  $[x_A^*, x_B^*(v_j)]$ .

Observe that the effort of  $B$  maximizes

$$\frac{x_B}{x_B + x_A^*} v_j - x_B.$$

The first-order condition gives

$$x_B(v_j) = \begin{cases} \sqrt{v_j} \sqrt{x_A} - x_A & \text{for } \sqrt{v_j} - \sqrt{x_A} > 0 \\ 0 & \text{for } \sqrt{v_j} - \sqrt{x_A} \leq 0 \end{cases}, \quad (1)$$

from which it is apparent that some low  $v_B$  types may not participate actively in the contest.

For now, fix a distribution  $\mu \in P^N$  of player  $B$  types and a set of inactive types  $1, \dots, k$  (i.e.,  $x_B(v_j) = 0$  for  $j = 1, \dots, k$ ), whilst  $k + 1, \dots, N$  participate actively (i.e.,  $x_B(v_j) > 0$  for  $j = k + 1, \dots, N$ ); if  $k = 0$ , then all player  $B$  types exert effort. When  $k > 0$ , player  $A$  wins with certainty if he meets types  $1, \dots, k$ , making his expected payoff

$$\left( \sum_{h=1}^k \mu_h + \sum_{m=k+1}^N \frac{\mu_m x_A}{x_A + x_B(v_m)} \right) v_A - x_A.$$

The first-order condition is

$$\left( \sum_{m=k+1}^N \frac{\mu_m x_B(v_m)}{(x_A + x_B(v_m))^2} \right) v_A = 1. \quad (2)$$

Solving (1) and (2) gives a solution for  $x_A$  when  $k$  types are inactive as

$$x_A^*(k) = \left( \frac{\sum_{m=k+1}^N \left( \frac{\mu_m}{\sqrt{v_m}} \right)}{\frac{1}{v_A} + \sum_{m=k+1}^N \left( \frac{\mu_m}{v_m} \right)} \right)^2. \quad (3)$$

Replacing  $x_A^*$  in (1) by (3) gives

$$x_B^*(v_j) = \sqrt{v_j} \left( \frac{\sum_{m=k+1}^N \left( \frac{\mu_m}{\sqrt{v_m}} \right)}{\frac{1}{v_A} + \sum_{m=k+1}^N \left( \frac{\mu_m}{v_m} \right)} \right) - \left( \frac{\sum_{m=k+1}^N \left( \frac{\mu_m}{\sqrt{v_m}} \right)}{\frac{1}{v_A} + \sum_{m=k+1}^N \left( \frac{\mu_m}{v_m} \right)} \right)^2, \quad j = k + 1, \dots, N. \quad (4)$$

None of the inactive player  $B$  types will want to exert positive effort as long as  $\sqrt{v_k} - \sqrt{x_A^*(k)} \leq 0$ . Using (3) and (4) yields total effort with  $k$  inactive types,  $TE(\mu, k)$ , as

$$TE(\mu, k) = x_A^*(k) + \sum_{m=k+1}^N \mu_m x_B^*(v_m). \quad (5)$$

Zhang and Zhou [2016] consider an internal solution, in which case  $k = 0$  and the total expected effort is

$$TE(\mu, 0) = \frac{E_{\mu^s}[\sqrt{v_B}] E_{\mu^s}\left[\frac{1}{\sqrt{v_B}}\right]}{\frac{1}{v_A} + E_{\mu^s}\left[\frac{1}{v_B}\right]}. \quad (6)$$

The expression in (1) makes it clear that low  $v_B$  types may not find it profitable to exert effort. This implies that participation has to be checked for player  $B$  of lowest type  $v_1$  first, given that the other players exert positive effort. Only if type  $v_1$  makes a positive contest effort do we have the internal equilibrium of Zhang and Zhou [2016]; if type  $v_1$  does not exert effort, then active participation is checked for  $v_2$  given that all types with a higher valuation participate. This proceeds in sequence until two adjacent types are identified such that  $x_B^*(v_k) = 0, x_B^*(v_{k+1}) > 0$ . Lemma 1 determines the set of active types for a given  $\mu \in P^N$ .

**Lemma 1.** *Consider  $\mu \in P^N$ . There exists thresholds  $\theta_k(\mu) > 0, k \in \{1, \dots, N-1\}$  where  $\theta_k(\mu) \leq \theta_{k+1}(\mu)$  for all  $k$  and with strict inequality if  $\max\{\mu_{k+1}, \dots, \mu_N\} > 0$ , such that  $\theta_k(\mu) \leq v_A < \theta_{k+1}(\mu)$  yields  $x_B^*(v_j) = 0$ , for  $j \in \{1, \dots, k\}$  and  $x_B^*(v_j) > 0$ , for  $j \in \{k+1, \dots, N\}$ .*

*Proof.* Suppose that player  $B$  types  $j = 1, \dots, k$  set  $x_B^*(v_j) = 0$ . From (1), type  $k$  will not want to change action if  $\sqrt{v_k} \leq \sqrt{x_A^*(k)}$ , i.e.,

$$\sqrt{v_k} \leq \frac{\sum_{m=k+1}^N \left(\frac{\mu_m}{\sqrt{v_m}}\right)}{\frac{1}{v_A} + \sum_{m=k+1}^N \left(\frac{\mu_m}{v_m}\right)}, \quad (7)$$

which reduces to

$$v_A \geq \frac{\sqrt{v_k}}{\sum_{m=k+1}^N \frac{\mu_m(\sqrt{v_m} - \sqrt{v_k})}{v_m}} := \theta_k(\mu). \quad (8)$$

Type  $k$  being inactive, it follows from (7) that player  $B$  types with  $v_j < v_k$  will not participate if  $x_B^*(v_k) = 0$ . By construction, player  $B$  types with  $v_j > v_k$  will participate if  $v_A < \theta_{k+1}(\mu)$ . To see  $\theta_k(\mu) \leq \theta_{k+1}(\mu)$ , note that for any  $m > k$ ,

$$v_k < v_{k+1} \Rightarrow \frac{\mu_m(\sqrt{v_m} - \sqrt{v_{k+1}})}{v_m\sqrt{v_{k+1}}} \leq \frac{\mu_m(\sqrt{v_m} - \sqrt{v_k})}{v_m\sqrt{v_k}}. \quad (9)$$

Summing (9) over  $m \in \{k+1, \dots, N\}$ ,

$$\begin{aligned} \sum_{m=k+2}^N \frac{\mu_m(\sqrt{v_m} - \sqrt{v_{k+1}})}{v_m\sqrt{v_{k+1}}} &\leq \sum_{m=k+1}^N \frac{\mu_m(\sqrt{v_m} - \sqrt{v_k})}{v_m\sqrt{v_k}} \\ \Rightarrow \frac{1}{\theta_{k+1}(\mu)} &\leq \frac{1}{\theta_k(\mu)} \Rightarrow \theta_k(\mu) \leq \theta_{k+1}(\mu). \end{aligned} \quad (10)$$

The inequality in (10) holds strictly if  $\max\{\mu_{k+1}, \dots, \mu_N\} > 0$ , in which case,  $\theta_k(\mu) < \theta_{k+1}(\mu)$ .  $\square$

Setting  $\theta_0(\mu) = 0$  and  $\theta_N(\mu) = \infty$ , by Lemma 1, we can express the equilibrium total effort for a given belief  $\mu \in P^N$  as

$$TE^e(\mu) = TE(\mu, k) \text{ if } v_A \in [\theta_k(\mu), \theta_{k+1}(\mu)], k = 0, 1, \dots, N-1. \quad (11)$$

For the following analysis, it will be useful to work out belief-free thresholds, which are outlined in Lemma 2.

**Lemma 2.** Fix  $k \in \{1, \dots, N-1\}$ . Denote  $\min_{\mu \in P^N} \theta_k(\mu)$  by  $\theta_k^{min}$ . Then,

$$\theta_k^{min} = \min_{v_m \in \{v_{k+1}, \dots, v_N\}} \frac{v_m \sqrt{v_k}}{\sqrt{v_m} - \sqrt{v_k}}.$$

Further,  $\theta_k^{min} < \theta_{k+1}^{min}$ .

*Proof.*  $\theta_k$  is minimized by identifying the largest value of  $\frac{(\sqrt{v_m} - \sqrt{v_k})}{v_m}$  for  $m = k+1, \dots, N$ , and attaching belief 1 to this particular  $v_m$  and zero to all others. To show that  $\theta_k^{min} < \theta_{k+1}^{min}$ , first note that  $\frac{v_m \sqrt{v_k}}{\sqrt{v_m} - \sqrt{v_k}}$  is increasing in  $v_k$ . Therefore, for any  $m \in \{k+2, \dots, N\}$ ,  $\frac{v_m \sqrt{v_{k+1}}}{\sqrt{v_m} - \sqrt{v_{k+1}}} > \frac{v_m \sqrt{v_k}}{\sqrt{v_m} - \sqrt{v_k}}$  for a common  $v_m$ . Suppose that  $v_M \in \{v_{k+2}, \dots, v_N\}$  minimizes  $\theta_{k+1}^{min} = \frac{v_M \sqrt{v_{k+1}}}{\sqrt{v_M} - \sqrt{v_{k+1}}}$ . Then it is possible to choose the same  $v_M$  and reach a lower value of  $\theta_k^{min}$ . Hence  $\theta_k^{min} < \theta_{k+1}^{min}$  for  $k \in \{1, \dots, N-1\}$ .  $\square$

Note that if  $v_A < \theta_1^{min}$ , then for any posterior belief  $\mu^s$ , all player  $B$  types participate actively in the contest and the internal solution of Zhang and Zhou [2016] is valid. In general, for  $v_A < \theta_k^{min}$ , there will be no set of beliefs such that all player  $B$  types  $1, \dots, k$  will be inactive simultaneously.

Following Kamenica and Gentzkow [2011], we can determine the optimal information disclosure from the concave closure of  $TE^e(\mu)$ . The principal increases her expected payoff to the concavification of  $TE^e(\mu)$  by optimally choosing a distribution of *Bayes-plausible* posteriors generated from the signaling distributions  $\{\Pr[m_s | v_j], m_s \in S, j \in \{1, \dots, N\}\}$ . If  $TE^e(\mu)$  is globally concave (convex), then no- (full-) information disclosure yields the principal a payoff same as the concavification of  $TE^e(\mu)$ . The principal's preferred signaling mechanism can partially disclose information only if  $TE^e(\mu)$  has both concave or convex parts. To highlight the role of information disclosure in the case of  $k = 0$  (all types participate actively), and  $k > 0$  (some inactive types), we first present the binary-type case and then look at the case of more types.

## 2.1 $N = 2$

Consider a posterior  $\mu = (\mu_1, \mu_2) \in P^2$  over player  $B$  types  $(v_1, v_2)$ . Since  $N = 2$ , the posterior  $\mu$  can be identified with a scalar  $\mu_2 = \Pr[v_B = v_2] \in [0, 1]$ . Both types exert effort in the contest for any  $\mu_2$  if  $v_A < \theta_1^{min} = \frac{v_2 \sqrt{v_1}}{(\sqrt{v_2} - \sqrt{v_1})}$ . Zhang and Zhou [2016, Lemma 1 and Proposition 3] show that the total effort  $TE(\mu, 0)$  with both player  $B$  types active is strictly concave in  $\mu_2 \in [0, 1]$  for  $v_A < \sqrt{v_2 v_1}$  and therefore no disclosure is optimal; and  $TE(\mu, 0)$  is strictly convex in  $\mu_2 \in [0, 1]$



for  $v_A > \sqrt{v_2 v_1}$  and therefore full disclosure is optimal.<sup>2</sup> Note that  $\theta_1^{min} > \sqrt{v_2 v_1}$ , and so the full-information disclosure finding of Zhang and Zhou [2016] holds for  $\sqrt{v_2 v_1} < v_A < \theta_1^{min}$ .

**Fact 1.** [Zhang and Zhou [2016, Proposition 3, modified]] For  $N = 2$ , consider  $v_A < \theta_1^{min}$ . Then, both types of player B exert non-zero effort in the contest under asymmetric information for any posterior  $\mu$ . Further, for  $v_A < \sqrt{v_2 v_1}$ , no disclosure is optimal and for  $\sqrt{v_2 v_1} < v_A < \theta_1^{min}$ , full disclosure is optimal.

This is an important result since Zhang and Zhou [2016] show that the general case with  $N > 2$  can be reduced to that of  $N = 2$ . For our extended parameter space, even the case  $N = 2$  is not so clear cut; we show below that partial information disclosure can be optimal.

Consider  $v_A \geq \theta_1^{min}$ . By Lemma 1 and the fact that  $\theta_1(\mu_2)$  is decreasing in  $\mu_2$ , there exists a unique  $\tilde{\mu}_2$  satisfying  $v_A = \theta_1(\tilde{\mu}_2)$  such that both types exert effort for  $\mu_2 \in [0, \tilde{\mu}_2)$ . Direct calculation gives

$$\tilde{\mu}_2 = \frac{v_2 \sqrt{v_1}}{v_A (\sqrt{v_2} - \sqrt{v_1})}.$$

For  $\mu_2 \in [\tilde{\mu}_2, 1]$ , type 1 is inactive and  $TE^e(\mu_2) = TE(\mu_2, 1)$ . We can calculate the derivatives as

$$\frac{\partial TE(\mu_2, 1)}{\partial \mu_2} = \frac{2\mu_2 v_A v_2^2 (v_A + v_2)}{(\mu_2 v_A + v_2)^3} > 0, \quad (12)$$

$$\frac{\partial^2 TE(\mu_2, 1)}{\partial \mu_2^2} = \frac{2v_A v_2^2 (v_2 - 2v_A \mu_2)}{(\mu_2 v_A + v_2)^4}. \quad (13)$$

Define  $\hat{\mu}_2 := \frac{v_2}{2v_A}$ . From (12) and (13), it follows that  $TE(\mu_2, 1)$  is always increasing in  $\mu_2$ , strictly concave (convex) for  $\mu_2 > (<) \hat{\mu}_2$ . When  $\hat{\mu}_2 \geq 1$ , which occurs if  $v_A \leq \frac{v_2}{2}$ , the total expected effort is piecewise convex in  $\mu_2$ . Lemma 3 shows that full information disclosure remains optimal.

**Lemma 3.** Suppose  $\theta_1^{min} < \frac{v_2}{2}$  and consider  $v_A \in [\theta_1^{min}, \frac{v_2}{2}]$ . Then, full information disclosure is optimal.

*Proof.* Note that  $TE^e(\mu_2)$  is given by  $TE(\mu_2, 0)$  for  $\mu_2 \in [0, \tilde{\mu}_2)$ , and  $TE(\mu_2, 1)$  otherwise; both functions are convex in  $\mu_2$  and  $TE^e(\mu_2)$  is continuous at  $\tilde{\mu}_2$ . Therefore,  $TE^e(\mu_2)$  is continuous and piecewise convex in  $\mu_2 \in [0, 1]$ . Further,

$$\begin{aligned} TE^e(\tilde{\mu}_2) &= TE(\tilde{\mu}_2, k=0) \leq (1 - \tilde{\mu}_2) TE(\mu_2=0, k=0) + \tilde{\mu}_2 TE(\mu_2=1, k=0) \\ &= (1 - \tilde{\mu}_2) TE(\mu_2=0, k=0) + \tilde{\mu}_2 TE(\mu_2=1, k=1) \\ &= (1 - \tilde{\mu}_2) TE^e(0) + \tilde{\mu}_2 TE^e(1), \end{aligned}$$

which follows from convexity of  $TE(\mu_2, 0)$  and the fact that  $TE(\mu_2=1, k=0) = TE(\mu_2=1, k=1) = \frac{v_A v_2}{v_A + v_2}$ . Therefore, the graph of  $TE^e(\mu_2)$  will always be lower than the straight line joining  $TE^e(0)$  and  $TE^e(1)$ , implying that full disclosure is optimal.  $\square$

<sup>2</sup>Unlike us, Zhang and Zhou [2016] describe concavity/convexity property of (6) in terms of  $\mu_1 = \Pr[v_B = v_1]$ . However, the findings are comparable since the second-order derivatives of  $TE^e$  with respect to  $\mu_1$  and  $\mu_2 = (1 - \mu_1)$  have the same sign.

When  $\hat{\mu}_2 < 1$ , which occurs if  $v_A > \frac{v_2}{2}$ , total expected effort is concave for  $\mu_2 \geq \max\{\hat{\mu}_2, \tilde{\mu}_2\}$  and either convex or piecewise convex for  $\mu_2 < \max\{\hat{\mu}_2, \tilde{\mu}_2\}$ . Proposition 1 shows that the partial information disclosure is optimal for sufficiently large values of  $v_A$ .

**Proposition 1.** *Consider  $v_A > \max\{\theta_1^{min}, \frac{v_2}{2}\}$ . Then, there exists  $\bar{v}_A > \max\{\theta_1^{min}, \frac{v_2}{2}\}$  such that  $\max\{\theta_1^{min}, \frac{v_2}{2}\} < v_A < \bar{v}_A$ , full information disclosure is optimal and for  $\bar{v}_A \leq v_A$ , partial information disclosure is optimal.*

*Proof.*  $TE^e(\mu_2)$  is given by  $TE(\mu_2, 0)$  for  $\mu_2 \in [0, \tilde{\mu}_2)$ , and  $TE(\mu_2, 1)$  for  $\mu_2 \in [\tilde{\mu}_2, 1]$ ; the former is convex, whilst the latter is either concave for  $\mu_2 \in [\tilde{\mu}_2, 1]$  if  $\hat{\mu}_2 \leq \tilde{\mu}_2$ , or, first convex for  $\mu_2 \in [\tilde{\mu}_2, \hat{\mu}_2]$  and then concave for  $\mu_2 \in [\hat{\mu}_2, 1]$  if  $\tilde{\mu}_2 < \hat{\mu}_2$ . Full information disclosure is optimal if  $(1 - \mu_2)TE^e(0) + \mu_2TE^e(1) = (1 - \mu_2)\frac{v_A v_1}{v_A + v_1} + \mu_2\frac{v_A v_2}{v_A + v_2} > TE^e(\mu_2)$  for all  $\mu_2 \in (0, 1)$ ; necessary and sufficient for that is the slope of the straight line is greater than the slope of  $TE^e(\mu_2)$  measured at  $\mu_2 = 1$ , which requires

$$\frac{v_A v_2}{v_A + v_2} - \frac{v_A v_1}{v_A + v_1} > \frac{2v_A v_2^2}{(v_A + v_2)^2} \Leftrightarrow v_A^2(v_2 - v_1) - v_A v_2(v_1 + v_2) - 2v_1 v_2^2 < 0 \Leftrightarrow v_A < \bar{v}_A,$$

where  $\bar{v}_A = \frac{v_2}{2} \left[ \frac{v_1 + v_2 + \sqrt{v_2^2 + 10v_1 v_2 - 7v_1^2}}{v_2 - v_1} \right]$ . When  $v_A > \bar{v}_A$ , define  $\bar{\mu}_2$  that solves  $\frac{TE(\bar{\mu}_2, 1) - TE(0, 0)}{\bar{\mu}_2} = \frac{\partial TE(\mu_2, 1)}{\partial \mu_2} |_{\bar{\mu}_2}$ . The concavification of  $TE^e(\mu_2)$  consists of the line  $\left(\frac{\bar{\mu}_2 - \mu_2}{\bar{\mu}_2}\right)TE(0, 0) + \frac{\mu_2}{\bar{\mu}_2}TE(\bar{\mu}_2, 1)$  for  $\mu_2 \in [0, \bar{\mu}_2]$  and  $TE(\mu_2, 1)$  for  $\mu_2 \in [\bar{\mu}_2, 1]$ . Then the principal uses partial information disclosure for  $\mu_2 \in [0, \bar{\mu}_2]$  and no disclosure otherwise.  $\square$

Example 1 illustrates partial information disclosure.

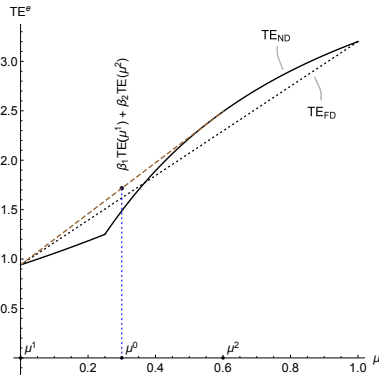


Figure 1:  $TE^e$  against  $\mu_2$ ,  $N = 2$

**Example 1.** Consider  $N = 2$ ,  $v_1 = 1$ ,  $v_2 = 4$ , and  $v_A = 16$ . In this case,  $\theta_1^{min} = \frac{v_2 \sqrt{v_1}}{\sqrt{v_2} - \sqrt{v_1}} = 4 < v_A$ , and  $\hat{\mu}_2 = \frac{v_2}{2v_A} = 0.125 < \tilde{\mu}_2 = \frac{v_2 \sqrt{v_1}}{v_A(\sqrt{v_2} - \sqrt{v_1})} = 0.25$ . Therefore, for  $\mu_2 < \tilde{\mu}_2$ , both types are active and  $TE^e(\mu)$  is convex; For  $\mu_2 \geq \tilde{\mu}_2$ , only type  $v_2$  is active and  $TE^e(\mu)$  is concave. Figure 1 plots  $TE^e(\mu)$  against  $\mu_2 \in [0, 1]$ . For  $\mu_2 = 0.3$ , the principal's payoffs from no disclosure and from full disclosure are 1.4876 and 1.61882, respectively. Consider a distribution of Bayes-plausible

posteriors:  $\mu^1 = (1, 0)$ ,  $\mu^2 = (0.4, 0.6)$  with probabilities  $\beta_1 = 0.5$ ,  $\beta_2 = 0.5$ , which can be generated with two messages  $m_1$  and  $m_2$  and the signal distributions matrix:

$$S = \begin{bmatrix} 5/7 & 2/7 \\ 0 & 1 \end{bmatrix},$$

where  $S_{(ij)}$  denotes  $\Pr[m_j | v_i]$ ,  $i \in \{1, 2\}$ ,  $j \in \{1, 2\}$ . From Kamenica and Gentzkow [2011], we know that the principal's payoff from partial disclosure of the above kind is  $\beta_1 TE(\mu^1) + \beta_2 TE(\mu^2) = 1.71626$ , which is higher than her payoffs from full or no disclosure.

## 2.2 $N \geq 3$

For  $N \geq 3$ , Zhang and Zhou [2016, Corollary 2] show that full disclosure is optimal for sufficiently high  $v_A$  (i.e.,  $v_A \geq \sqrt{v_{N-1}v_N}$ ), and partial disclosure can arise otherwise. For our extended parameter space, partial disclosure can be optimal even for high values of  $v_A$ . To understand why, recall the underlying mechanism in Zhang and Zhou [2016]: For  $\mu \in \text{int}(P^N)$ , there always exists a direction along which  $TE(\mu, 0)$  is convex, and therefore, the principal can obtain a higher expected payoff from a distribution over two Bayes-plausible posteriors on  $\text{Edge}(P^N)$  where the directional vector intersects  $\text{Edge}(P^N)$ . This reduces dimension of the problem by one, and gradually optimal posteriors can be found on pairwise edges. The analysis of  $N = 2$  case shows that these edges are fully convex (concave) for high (low) values of  $v_A$  when only interior solutions are considered. However, as we have shown, the possibility of corner solution implies that pairwise edges will not always be convex for high  $v_A$ , because of which the findings of Zhang and Zhou [2016] will not hold.<sup>3</sup> Example 2 illustrates how partial disclosure can dominate full or no disclosure.

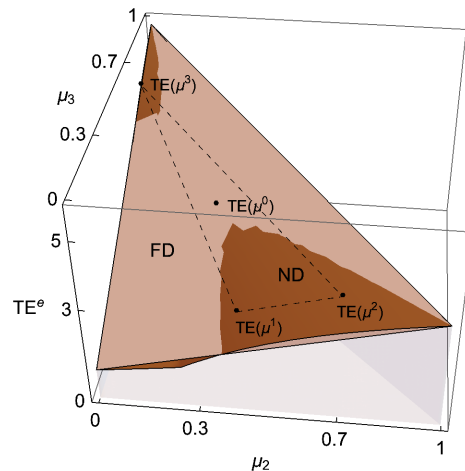


Figure 2:  $TE^e$  against  $(\mu_2, \mu_3)$ ,  $N = 3$

<sup>3</sup>In addition, we conjecture that the finding that  $TE^e$  is convex along “some” directional vector for  $\mu \in \text{int}(P^N)$ , which holds when all types are active, is not robust when some types choose to remain inactive. Therefore, the optimal posteriors may not necessarily be found on  $\text{Edge}(P^N)$ .

**Example 2.** Consider  $N = 3$ ,  $v_1 = 1$ ,  $v_2 = 4$ ,  $v_3 = 9$ , and  $v_A = 16$ . We have  $\theta_1^{min} = \min \left\{ \frac{v_2\sqrt{v_1}}{\sqrt{v_2}-\sqrt{v_1}}, \frac{v_3\sqrt{v_1}}{\sqrt{v_3}-\sqrt{v_1}} \right\} = 4$ ,  $\theta_2^{min} = \frac{v_3\sqrt{v_2}}{\sqrt{v_3}-\sqrt{v_2}} = 18$ , and  $\theta_1^{min} < v_A < \theta_2^{min}$ . Further,  $\theta_1(\mu) = \frac{36}{9\mu_2+8\mu_3}$  and  $v_A < \theta_1(\mu) \Leftrightarrow 36\mu_2 + 32\mu_3 < 9$ . Therefore, for prior  $\mu^0$  with  $36\mu_2^0 + 32\mu_3^0 < 9$ , all three types are active and for  $\mu^0$  with  $36\mu_2^0 + 32\mu_3^0 \geq 9$ , the type  $v_1$  will be inactive. Figure 2 plots  $TE^e$  against  $(\mu_2, \mu_3)$ ,  $0 \leq \mu_2 + \mu_3 \leq 1$ .  $TE^e$  is neither globally concave or convex. The darker region on top of the graph represents the area where the principal's payoffs from no disclosure is higher than that from full disclosure. For  $\mu^0 = (0.3, 0.3, 0.4)$ , her payoffs from full and no disclosure are 3.54635 and 3.53056, respectively. Consider a distribution of Bayes-plausible posteriors:  $\mu^1 = (0.5, 0.4, 0.1)$ ,  $\mu^2 = (0.2, 0.7, 0.1)$ ,  $\mu^3 = (0.2, 0, 0.8)$  with probabilities  $\beta_1 = 0.333333$ ,  $\beta_2 = 0.238095$ ,  $\beta_3 = 0.428571$ , which can be generated with three messages  $m_1, m_2, m_3$ , and the signal distributions matrix:

$$S = \begin{bmatrix} 0.555556 & 0.15873 & 0.285714 \\ 0.444444 & 0.555556 & 0 \\ 0.083333 & 0.059524 & 0.857143 \end{bmatrix},$$

where  $S_{(i,j)} = \Pr[m_j | v_i]$ . The principal's payoff from partial disclosure is  $\beta_1 TE(\mu^1) + \beta_2 TE(\mu^2) + \beta_3 TE(\mu^3) = 3.60892$ , which is higher than her payoffs from full or no disclosure. Although the posteriors considered here are not necessarily optimal, the exercise shows that the payoff from partial disclosure can dominate that from full or no disclosure.

## References

- Gil S Epstein and Yosef Mealem. Who gains from information asymmetry? *Theory and decision*, 75(3):305–337, 2013. DOI: <https://doi.org/10.1007/s11238-013-9351-x>.
- Emir Kamenica and Matthew Gentzkow. Bayesian persuasion. *American Economic Review*, 101(6):2590–2615, 2011. DOI: <https://doi.org/10.1257/aer.101.6.2590>.
- Jun Zhang and Junjie Zhou. Information disclosure in contests: A bayesian persuasion approach. *The Economic Journal*, 126(597):2197–2217, 2016. DOI: <https://doi.org/10.1111/eoj.12277>.